Grade 11 June Exam P1 Solns

SIT/file

1.1.1 $3x^2 = 5x$

$3x^2 - 5x = 0$ ✓ std

$x(3x-5) = 0$ ✓ fact

$x = 0$ or $x = \frac{5}{3}$ ✓ both (3)

1.1.2 $3x^2 - 2x - 6 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\quad \frac{2 \pm \sqrt{76}}{6}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(3)(-6)}}{2(3)}$ ✓ subt f

$x = -1,12$ ✓ or $x = 1,79$ ✓ (3)

1.1.3 $2\sqrt{x+6} + 2 = x$

$2\sqrt{x+6} = x-2$ ✓ isolate (4)

$(2\sqrt{x+6})^2 = (x-2)^2$ ✓ squaring both sides

$4(x+6) = x^2 - 4x + 4$ ✓

$x^2 - 8x - 20 = 0$

$(x-10)(x+2) = 0$

$x = 10$ or $x \neq -2$ ✓ ans + selection

1.1.4 $x^2 < -2x + 15$ $\quad \div -1$

$x^2 + 2x - 15 < 0$ ✓ std

$(x-3)(x+5) < 0$ ✓ factors (4)

$x = 3$ or $x = -5$

+ 0 − 0 +

-5 3

$-5 < x < 3$. ✓✓ A ie 2 or 0 no CA

1.1.5 $2^{x+4} + 2^x = 8704$

$2^x \cdot 2^4 + 2^x = 8704$

$2^x(2^4+1) = 8704$ ✓

$2^x = \frac{8704}{17}$

$2^x = 512$ ✓

$2^x = 2^9$ or $x = \frac{\log 512}{\log 2}$

$x = 9$ ✓ (3)

1.1.6 $3x^2 = 7$

$x^2 = \frac{7}{3}$ ✓ isolate

$x = \pm\sqrt{\frac{7}{3}}$ ✓ $\pm\sqrt{\ }$ (3)

$x = \pm 1{,}53$ ✓ ans

1.1.7 $\frac{6^{2x}}{9^x} = \sqrt[3]{4}$

pb ✓ $(2.3)^{2x} \cdot 3^{-2x} = 4^{\frac{1}{3}}$ ✓ rational exp

$2^{2x} \cdot 3^{2x} \cdot 3^{-2x} = 2^{\frac{2}{3}}$

$2^{2x} = 2^{\frac{2}{3}}$ ✓

$2x = \frac{2}{3}$ (4)

$x = \frac{1}{3}$ ✓

1.2. $3^{x+y} = 27$ and $x^2 + y^2 = 17$

$3^{x+y} = 3^3$

$x + y = 3$ ✓ $\Rightarrow y = 3 - x$ ✓

$x^2 + (3-x)^2 = 17$ ✓

$2x^2 - 6x - 8 = 0$ (÷ 2)

$x^2 - 3x - 4 = 0$ ✓

$(x-4)(x+1) = 0$ ✓ (7)

$x = 4$ or $x = -1$ ✓

$y = -1$ or $y = 4$ ✓

1.3 $a^2 - 5ab + 4b^2 = 0$

$(a-b)(a-4b) = 0$ ✓

$a = b$ ✓ or $a = 4b$ ✓ (3)

1.4 $A = (1-a)$

$B = (1+a)(1+a^2)(1+a^4) \ldots (1+a^{512})$

$A \times B = (1-a)(1+a)(1+a^2)(1+a^4) \ldots (1+a^{512})$

$= (1-a^2)(1+a^2)(1+a^4) \ldots (1+a^{512})$ ✓

$= (1-a^4)(1+a^4) \ldots (1+a^{512})$

$= (1-a^{512})(1+a^{512})$ ✓

$= \underline{1-a^{1024}}$ ✓ (3)

Question 2

2.1 $x(2x-5)+2 = 0$

$2x^2 - 5x + 2 = 0$

$a = 2 \quad b = -5 \quad c = 2$

$\Delta = b^2 - 4ac$

$\Delta = (-5)^2 - 4(2)(2)$ ✓ sub in

$= 25 - 16$

$= \underline{9}$ ✓ Δ (3)

∴ roots are real, rational and unequal. ✓

2.2 $x^2 + px = 40$

$x^2 + px = 40$

$(-5)^2 - 5p = 40$ ✓

$25 - 5p = 40$

$-5p = 15$

$p = -3$ ✓

other root

$x^2 - 3x - 40 = 0$

$(x-8)(x+5) = 0$

$x = 8$ or $x = -5$

∴ other root

$\underline{x = 8}$ ✓

(3)

2.3.1 $x^2 + 2kx = -9k - 4x$

$x^2 + 2kx + 4x + 9k = 0$ ✓ (1)

$x^2 + x(2k+4) + 9k = 0$

2.3.2 Real roots $\Delta \geq 0$

$a = 1 \quad b = 2k+4 \quad c = 9k$

$\Delta = b^2 - 4ac$

$\Delta = (2k+4)^2 - 4(1)(9k)$ ✓ sub in

$= 4k^2 + 16k + 16 - 36k$

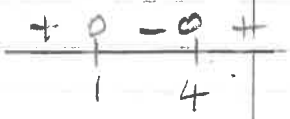
$= 4k^2 - 20k + 16$

but $\Delta \geq 0$ ✓ $\Delta \geq 0$ (4)

$4k^2 - 20k + 16 \geq 0 \quad \div 4$

$k^2 - 5k + 4 \geq 0$

$(k-4)(k-1) \geq 0$



C.V $k = 4$ or $k = 1$

∴ $k \leq 1$ or $4 \leq k$ ✓ both

2.4 $x - 1 = \frac{2}{mx}$

$mx(x-1) - 2 = 0$

$mx^2 - mx - 2 = 0$ ✓ std

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (2)

$x = \frac{-(-m) \pm \sqrt{(-m)^2 - 4(m)(-2)}}{2(m)}$ ✓ sub in

$= \frac{m \pm \sqrt{m^2 + 8m}}{2m}$

Question 3 WOC

3.1 $27^{\frac{2}{3}} \cdot 81^{\frac{1}{2}}$

$= 3^{3(\frac{2}{3})} \cdot 3^{4(\frac{1}{2})}$ ✓ pb

$= 3^2 \cdot 3^2$ ✓ simplify

$= 81$ ✓ answ (3)

3.2 $\dfrac{64^{-\frac{2}{3}} \cdot \sqrt{8}}{\sqrt[7]{128} \cdot \sqrt{98}}$

$= \dfrac{2^{6(-\frac{2}{3})} \cdot \sqrt{4 \times 2}}{(2^7)^{\frac{1}{7}} \cdot \sqrt{49 \times 2}}$ ✓ num ✓ deno

$= \dfrac{2^{-4} \cdot 2\sqrt{2}}{2 \cdot 7\sqrt{2}}$

$= \dfrac{1}{16.7}$ ✓

$= \dfrac{1}{112}$ ✓ (4)

3.3 $\dfrac{3^{x-1} \cdot 3^{2x} - 3^{3x}}{27^x - 3^{3x+2}}$

$= \dfrac{3^x \cdot 3^{-1} \cdot 3^{2x} - 3^{3x}}{3^{3x} - 3^{3x} \cdot 3^2}$

$= \dfrac{3^{3x} \cdot 3^{-1} - 3^{3x}}{3^{3x} - 3^{3x} \cdot 3^2}$

$= \dfrac{3^{3x}(3^{-1} - 1)}{3^{3x}(1 - 3^2)}$ ✓ ✓ (4)

$\dfrac{\frac{1}{3} - 1}{1 - 9}$

$= \dfrac{-\frac{2}{3}}{-8}$ ✓ Simpl both num + den

$-8 = -\frac{8}{1}$

$= -\frac{2}{3} \times -\frac{1}{8}$

$= \frac{2}{24}$

$= \frac{1}{12}$ ✓

4. $\dfrac{15.5^x.5^{-1}+5^x.5}{2^{-x}}$

$= \dfrac{5^x(15.5^{-1}+5)}{2^{-x}}$ ✓ c.f

$= \dfrac{5^x\left(\frac{15}{5}+5\right)}{2^{-x}}$ ✓ Simpl

$= 5^x(8)2^x$ ✓

$= (2.5)^x\ 8$

$= 10^x.\ 8$

$= 8^m$

4 ~~3~~

3.5 $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \dfrac{1}{\sqrt{99}+\sqrt{100}}$

✓ rationalising

$= \dfrac{1}{\sqrt{1}+\sqrt{2}} \times \dfrac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}} + \cdots + \dfrac{1}{\sqrt{99}+\sqrt{100}} \times \dfrac{\sqrt{99}-\sqrt{100}}{\sqrt{99}-\sqrt{100}}$

$= -1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+2 \cdots -\sqrt{99}+10$

✓ Simplify

$= -1+10$

$= 9$ ✓ answ

(3)

Question 4

4.1 Seq A : 4 ; 11 ; 18 ; 25 , ...

Seq B : 11 ; 7 ; 3 , −1 , ...

4.1.1 32 ; 39. ✓ both (1)

4.1.2a) $T_n = a+(n-1)d$

$T_n = 4+(n-1)(7)$

$= 7n-3$. ✓ (1)

b) $T_n = 11+(n-1)(-4)$

$= -4n+15$ ✓ (1)

4.1.3 Show $U_{n+1} = U_n + 3$

$U_n = 7n-3+(-4n+15)$

$= 7n-3-4n+15$

$= 3n+12$ ✓

U_{n+1}

$= 3(n+1)+12$

$= 3n+3+12$

$= 3n+15$ ✓

$U_n + 3$

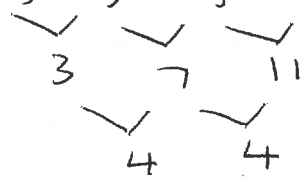
$= 3n+12+3$

$= 3n+15$ ✓

(3)

$\therefore U_{n+1} = U_n + 3$

4.2.1 2; 5; 12; 23; . . . ; 1277



(4)

$2a = 4$
$a = 2$ ✓

$3a + b = 3$
$3(2) + b = 3$ ✓
$b = -3$

$a + b + c = 2$
$2 - 3 + c = 2$ ✓
$c = 3$

$\therefore T_n = 2n^2 - 3n + 3$ ✓

4.2.2. $T_n = 2n^2 - 3n + 3$

$1277 = 2n^2 - 3n + 3$ ✓ equate

$2n^2 - 3n - 1274 = 0$ ✓ for 4.

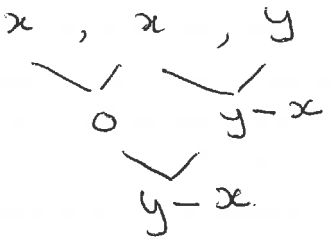
$(n - 26)(2n + 49) = 0$

(3)

$n = 26$ or $n \neq -\frac{49}{2}$

$\therefore$ 26 terms ✓ answer with selection

4.3 x , x , y



$x + x + y = 14$

$2x + y = 14$ --- (1) ✓ $S_3 = 14$

$y - x = 5$ -- (2) ✓ $d_2 = 5$

$y = x + 5$

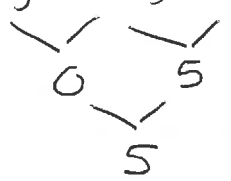
$2x + x + 5 = 14$
$3x = 14 - 5$
$3x = 9$
$x = 3$

(4)

$y = 3 + 5$

$y = 8.$

3; 3; 8



$\therefore T_1 = T_2 = 3$ ✓

$T_3 = 8.$ ✓

Question 5

5.1 $m_{PQ} = \frac{5-1}{-4-4}$ ✓ $P(-4;5)$ $Q(4;1)$

$= \frac{-1}{2}$ ✓ (2)

5.2 $\left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$ R $(-1;-4)$ M $(2;2)$ S $(x;y)$

$\frac{-1+x}{2} = 2$ $\quad \frac{-4+y}{2} = 2$

$x = 5$ ✓ ✓ $\quad y = 8$ (2)

$S(5;8)$

5.3 $m_{RMS} = \frac{8-(-4)}{5-(-1)}$ R $(-1;-4)$ S $(5;8)$ M $(2;2)$

OR $m_{RM} = \frac{2-(-4)}{2-(-1)} = \frac{12}{6}$

$= 2$ ✓

$\tan\theta = 2$

Ref$^\wedge$ $= 63,43...^\circ$ ✓ Statement

tan + in

I: $\theta = 63,43^\circ$ ✓ (3)

5.4.1 $m_{RM} = \frac{-4-2}{-1-2}$

$= 2.$

$m_{PQ} \times m_{RM} = -\frac{1}{2} \times 2$ ✓ ← product of m's; sub m's

$2/3 \hookrightarrow = -1$ ✓ (3)

$\therefore PM \perp RS.$

$\therefore P\hat{M}S = 90^\circ$

5.4.2 $PM = \sqrt{(-4-2)^2 + (5-2)^2}$ $\sqrt{45}$ $P(-4;5)$ $M(2;2)$

$= 3\sqrt{5}$ ✓

$RS = \sqrt{(5-1)^2 + (8+4)^2}$ $\sqrt{180}$ $R(-1;-4)$ $S(5;8)$ (4)

$= 6\sqrt{5}$ ✓

Area $\Delta PRS = \frac{1}{2}(6\sqrt{5})(3\sqrt{5})$ ✓ ±bh

$= 45$ sq units ✓

5.5. $RQ = \sqrt{(-1-4)^2 + (-4-1)^2}$ R(-1;-4) Q(4;1)

$= \sqrt{50}$

$= 5\sqrt{2}$ ✓

$SQ = \sqrt{(5-4)^2 + (8-1)^2}$ S(5;8) Q(4;1)

$= \sqrt{50}$ ✓

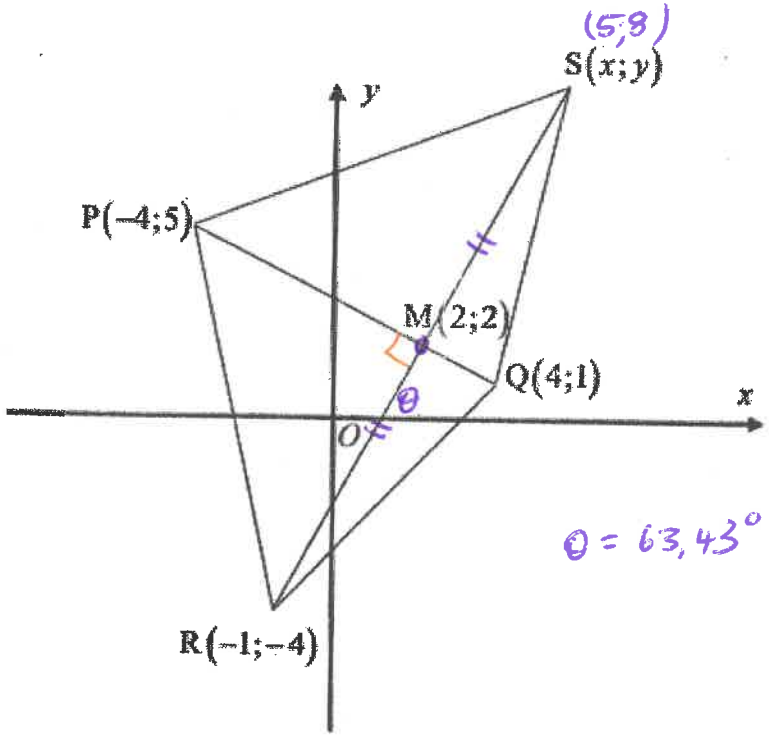
$= 5\sqrt{2}$ (2)

∴ ΔQRS is isosceles because

$RQ = SQ$

ie a Δ with 1 pr of sides =

5.



$\theta = 63,43°$